

Things that depend on frequency, and how to calculate them

We are going to talk about things that depend on frequency. This will include wavelength, reactance, impedance, insertion loss, return loss and SWR. Learning how to calculate things leads to greater insight into what these things mean and how their values affect our ability to operate our stations.

Let us start with wavelength, frequency, and the speed of light. We commonly measure the wavelength of our signals in meters. A meter is just a bit longer than a yard; more precisely it is 39.37 inches. For more than half of the twentieth century frequency was measured in "cycles per second". In 1960 the General Conference on Weights and Measures decided to rename "cycles per second" as Hertz in honor of Heinrich Hertz, a 19th century German physicist. The speed of light as every school child knows is about 186,000 miles per second. That number is easy to remember but incompatible wavelength in meters. We need the speed of light in kilometers per second. As it happens there are 1.609344 kilometers in each mile.

$$186,000 * 1.609344 = 299,338$$

This number is close enough to 300,000 kilometers per second and that is the number we use for most calculations. The actual formula we need for computing things is that the frequency of a radio wave times its wavelength is equal to the speed of light. In other words:

$$frequency(Hz.) * wavelength(meters) = 299,338,000(meters/second)$$

A common simplification is to express frequency in MHz. and express the speed of light as 300 megameters/second which lead to

$$frequency(MHz) * wavelength(meters) = 300(megameters/second)$$

Ohms law and DC Resistance

We remember Ohm's law that relates voltage (volts), current (amperes), and resistance (ohms). It looks like the following:

$$\text{Voltage}(\text{volts}) = \text{current}(\text{amperes}) * \text{resistance}(\text{ohms})$$

In AC circuits we have two new concepts associated with inductors and capacitors. These are called **inductive reactance**, and **capacitive reactance**. As you might have suspected both of these things depend on frequency.

The inductive reactance of an inductor (coil) is given by:

$$X_L = 2\pi f L$$

Here f is the frequency in Hertz and L is the inductance in henrys, named after a 19th century American scientist. You can see that the relationship is a linear one. That is the graph of inductive reactance as a function of frequency is a straight line. As the frequency goes up the inductive reactance goes up.

What does this all mean? At a frequency of 0, or DC, the inductive reactance is also 0. To DC the inductor might just as well be a piece of wire. As the frequency goes up it starts looking like a bigger and bigger impediment to the flow of current. At very high frequencies there will be no current flow at all because the inductive reactance is so large.

The capacitive reactance of a capacitor is given by:

$$X_C = \frac{1}{2\pi f C}$$

Here f is the frequency in Hertz and C is the capacitance in farads, named after Michael Faraday, a 19th century English physicist. This relationship is non-linear. That means the graph of capacitive reactance is not a straight line. So what is it? It is called a hyperbola and it looks like this:

Draw hyperbola on white board

What can we conclude from this picture? Well at very low frequencies, like DC, the capacitor

blocks all or most of the current flow because the reactance is so large. As we increase the frequency the reactance decreases dramatically at first and then more slowly. Eventually even with large increases in frequency the reactance hardly changes from a very low value. What does this mean? Well it means a capacitor will block DC current flow and at high frequencies it will pass current just like a short circuit.

What's with the 2π ?

You might be wondering about the factor of 2π that shows up in the reactance formula. Well it is used to convert frequency in Hertz, or cycles per second, or revolutions per second into a dimensionless angular measure called radians. Just like Noah asked, "What's a cubit" you might ask the same about radians. It is just a bit more than 57 degrees and there are 2π radians in a complete circle of 360 degrees. Periodic functions like radio waves and circles are intimately connected.

Skin Effect

As frequency increases, RF current flows in a thinner layer of the conductor, closer to the surface.

So what is impedance?

It is a combination of DC resistance which is not a function of frequency, and one or both types of reactance which are a function of frequency. There is a problem however. We cannot just combine them in a naive and simple fashion. Just like in grade school when the teacher said "You can't add apples and oranges", in radio you cannot just add resistance and reactance. What we need is a way of keeping track of all the information provided by the resistance number and the reactance number.

Cartesian Coordinates

The first idea is consider impedance as a pair of numbers with the first member of the pair being the resistance and the second number of the pair being the reactance. This idea of using an ordered pair of numbers to represent point in two dimensions was named after René Descartes, a 17th century French mathematician. For example if I have a resistance of 150 ohms and a reactance of 75 ohms I could write this as (150,75). Where is this going you ask. Well if we consider the points on the plane that are all the same distance from (0,0) we have a

semi-circle in the right half plane. We are restricted to the right half plane because there are no passive components with a negative resistance.

Another way of thinking about all the points on the semi-circle is to say they have the same magnitude. That is they all have the same distance from (0,0). How do we compute that magnitude or distance from (0,0)? It is the square root of the sum of the squares. Say what?

$$\sqrt{(R^2 + X^2)}$$

For the purposes of our example R squared is 22,500 and X squared is 5,625. The sum is 28,125 and the square root of that is 167.7 ohms. That wasn't so bad, most calculators have a single key that will do just that calculation.

Another piece of information contained in the pair of numbers is the angle between the x-axis (Resistance) and the line that contains the points (0,0) and (150,75). Fortunately most calculators have a single key function that will give us this answer. It is called the arctangent, \tan^{-1} , or inverse tangent. If the calculator expresses angle in degrees it works like this:

$$\tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{75}{150}\right) = 26.56^\circ = 0.4636 \text{ radians}$$

What happens if we have both types of reactance like we would in a series circuit? Well we can add them together with the following rule.

Inductive Reactance is positive

Capacitive Reactance is negative

A fun fact

The impedance of free space is about 377 ohms

Voltage across an inductor

Series RL circuit

Current through a capacitor

Series RC circuit

Resonance

Going back to our discussion of the two types of reactance ask what might happen in a circuit where the inductive reactance is equal to the capacitive reactance. We know from the rule above that the net reactance will be 0, so all that is left is the resistance. Since both types of reactance depend on frequency there can only be one unique frequency where this will be true. How can we find that frequency? We set the two expressions equal to each other

$$\begin{aligned}2\pi f_r L &= \frac{1}{2\pi f_r C} \\(2\pi f_r)^2 &= \frac{1}{LC} \\f_r &= \frac{1}{2\pi\sqrt{LC}}\end{aligned}$$

See graph of intersection of reactance functions

In a series resonant circuit the currents are the same in both components, but the voltages are 180° apart. In a parallel resonant circuit the voltages in both components are the same but the currents are 180° apart.

Filters

We use filters for two distinct purposes. The first is to pass signals of interest and the second is to block signals we are not interested in. Filters are frequency discriminating circuits. There are some basic types of passive filters that we can construct. They are:

1. Low Pass
2. High Pass
3. Bandpass
4. Notch

The response of a filter is usually displayed on Bode plot which shows the logarithm of the

magnitude of the output signal to the input signal. All passive filters have some insertion loss across a wide band of frequencies so the scale in dB is negative. In filters we are interested in the frequency at which the output signal is 3 dB below the input signal. That happens when the output signal is about 70% of the input signal. Who can explain the connection between 3 dB and 70%?

$$20\log\left(\frac{1}{\sqrt{2}}\right) = 20\log\left(\frac{1}{1.414}\right) = 20\log(.707) = 20(-0.1505) = -3.01$$

RF Filters

1. Bandpass
2. Trap

Transmission Lines

Antennas

the reflection coefficient rho is computed as the square root of reflected power over the forward power.

$$\rho = \sqrt{\frac{P_R}{P_F}}$$

$$\Gamma = \frac{V_r}{V_i} = \frac{Z_L - Z_S}{Z_L + Z_S}$$

SWR is calculated from the reflection coefficient as

$$SWR = \frac{1+\rho}{1-\rho} = \frac{1+\Gamma}{1-\Gamma}$$

Return loss is calculated from the reflection coefficient

$$RL(dB) = 20\log|\Gamma|$$

